Bob's TechTalk #27
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Capacitors - Part II of IV

Capacitor Voltage & Current

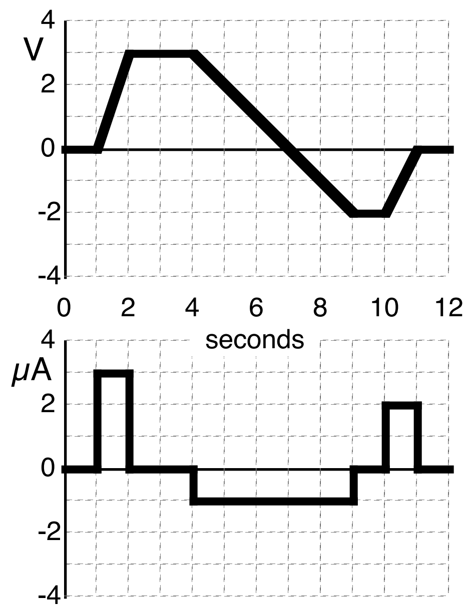
Last month we looked at the basic concept of a capacitor, how it stores energy in an electric field and various ways one can be constructed. This month we'll look at some uses for capacitors. but first we must learn the relationship between the voltage across a capacitor and the current flowing through it. As we learned last month, even thought a capacitor has no direct path between its leads, it still can carry a current. That relationship is shown by the following equation:

$$i = C\frac{dv}{dt}$$

This is what mathematicians call a differential equation. Luckily, you won't need to solve it, you'll just need to understand what it means, and that is not hard. In plain english, this equations says that *the current flowing through a capacitor is equal to the capacitance times the rate at which the voltage across the capacitor is changing with respect to time*. If the voltage is changing fast then the current is high. But if the voltage is changing slowly then the current is low. And, of course if the voltage isn't changing at all, then the current is zero.

Look at figure 1. The top graph shows the voltage across a 1µf capacitor over a 12 second period. The lower graph shows the current flowing through the capacitor:

- For the first second the voltage is zero but more importantly it is not changing, thus the current is zero.
- For the next second the voltage increases linearly at three volts per second, thus the current is 3 µA.
- Over the next two seconds the voltage remains constant at three volts and the current through the capacitor is again zero.
- Over the next five seconds the voltage is changing at minus one volt per second from plus three volts to minus two volts, and the current through the capacitor is -1 µA.
- For the next second the voltage is increasing by two volts per second and the current is 2 µA.
- Finally, for the twelfth second the voltage is again constant and the current is zero.



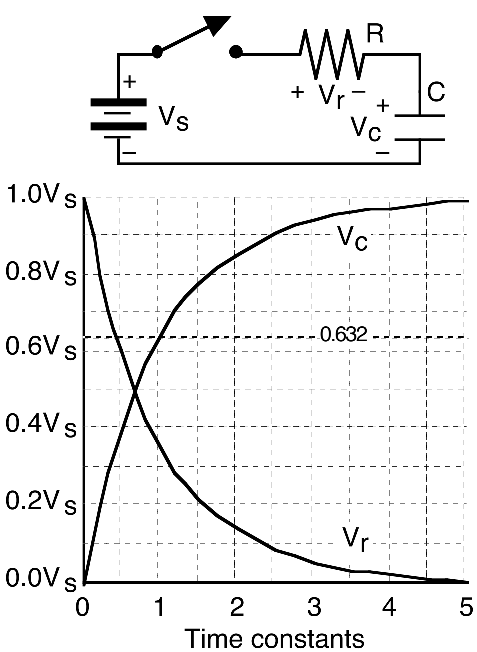
A varying voltage across a 1µf capacitor (top) and the resulting current through the capacitor
FIGURE 1

The important thing to remember here is that the current depends only on *the rate at which the voltage is changing* and not on the actual voltage itself.

Figure 2 shows a capacitor in series with a resistor. Assume that the capacitor is initially discharged. When the switch is thrown, current flows through the resistor charging the capacitor. The voltage across the capacitor increases exponentially until it is equal to the voltage of the battery, at which time no more current flows. For those who are interested, the voltage across the capacitor is given by the equation:

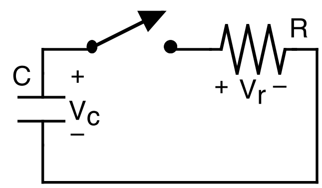
$$V_C = V_S\left(1 - e^{\left(\frac{-t}{RC}\right)}\right)$$

To some this may seem a complex equation, but all we need to know from it is that when $t = o$ then $V_C = 0$ because $e^o = 1$, and when $t = RC$ then $V_C = 0.632V_S$. RC is called one time constant and is the time it takes a capacitor to charge to 63.2%. After five time constants a capacitor is charged to greater than 99% and is considered fully charged. The voltage across the resistor and and capacitor add to Vs, the battery voltage.



Charging a capacitor. Voltages V_C is across the capacitor and V_r is across the resistor. $V_S = V_C + V_r$ **FIGURE 2**

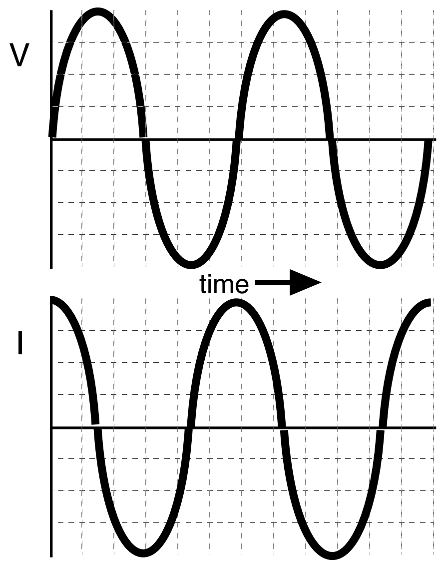
Discharge of a capacitor is also exponential. The circuit is shown in figure 3. Assume that the capacitor is initially charged to Vs. When the switch is closed, the capacitor starts to discharge through the resistor. After one RC time constant the voltage will have dropped to 0.368 of the original voltage. After 5 RC time constants the voltage will have dropped below 1% of the original voltage and and be considered fully discharged. Note that in the figure 3 Vr is equal to Vc and follows the same curve as Vr in the graph of figure 2.



Disharging a capacitor. Voltages V_C is across the capacitor and V_r is across the resistor. $V_C = V_r$ **FIGURE 3**

The charging and discharging a capacitor to a steady-state or DC condition represents a transient condition. Another condition, called a periodic condition occurs when a capacitor is subject to a periodic voltage or current. Periodic means that the voltage or current signal repeats over time. The time it takes to repeat is called the period and relates to frequency and wavelength. The sine wave is the most simple periodic wave.

Let's apply an AC sine wave voltage across a capacitor and see what happens to the current flowing through it. Figure 4 is similar to figure 1, except the applied voltage is a sine wave. The lower graph shows the resulting current through the capacitor. Notice that where the sine wave voltage is at a peak, its rate of change is momentarily zero, and the current is zero. Also, the points where the voltage crosses zero is where it is changing the fastest and the current is at a peak. The voltage and current are out of phase by 90° and the current is leading the voltage.



Sine wave voltage across a capacitor (top) and the resulting current through the capacitor
FIGURE 4

Remember back to Ohms law. The relationship between voltage and current in a resistor depends upon the resistance. In a capacitor the current is determined not by the voltage but by the capacitance and the rate at which the voltage is changing. For a sine wave, this rate is more commonly called the sine wave's frequency. Thus a capacitor has a characteristic similar to resistance for AC voltage. That characteristic is called *reactance*. We've all learned the formula for reactance of a capacitor as part of our ham test. It is:

$$X_C = \frac{1}{2\pi f C}$$

Note that as the capacitance gets larger or the frequency gets higher the reactance gets smaller. There are two things to note about capacitance. First, unlike a resistor, an **ideal capacitor doesn't dissipate any energy**. Second a **capacitor creates a phase shift** between the voltage and the current.

Next month we're going to explore the phase shift and get an understanding of power factor in AC circuits as well as in electrolytic capacitors.

73, from AF6C

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