

Bob's TechTalk #41 by Bob Eckweiler, AF6C

"L" Matching Network – Part II of II

Designing an 'L' Network

Introduction:

Last month the "L-Network" was introduced as a way to match two impedances at a particular frequency. Summarizing last month's article, a reactance is paralleled with the higher resistance impedance. The parallel circuit created has an equivalent series circuit. The parallel reactance is chosen so the resistance of the series circuit is the desired matching resistance. Finally a reactance of opposite magnitude is added in series to cancel the resulting series reactance. If that sounds confusing re-read part I.

There are two questions in last month's column that weren't addressed:

- ★ How are the component values chosen?
- ★ What are the pros and cons of the "L-Network"?

Calculating "L Network" Components:

Unlike last month, we'll have to break out the scratch paper, calculator and a sharp pencil! But the math is simple - I promise.

A term that was not mentioned last month is 'Q' or the 'quality factor' of the circuit. You can read more about 'Q' in the ARRL Handbook, but for us it is just an intermediate step for obtaining the needed component values.

Q for an "L-Network" is given by equation 1 *:

$$Q = \sqrt{\left(\frac{R_P}{R_S} - 1\right)} \quad \text{eq. 1.}$$

R_P and R_S are the two desired resistances to be matched so Q is easily calculated. Note that R_P must be the larger of the two values. From the example last month:

$$R_P = 50\Omega, \quad R_S = 36\Omega \quad \text{so}$$

$$Q = \sqrt{\left(\frac{50}{36} - 1\right)} = \sqrt{1.389 - 1}$$

$$Q = \sqrt{0.389} = 0.624$$

Use equation 2 to solve for X_P :

$$X_P = \frac{R_P}{Q} \quad \text{eq. 2.}$$

$$X_P = \frac{50\Omega}{0.624}$$

$$X_P = 80.18\Omega$$

And use equation 3 to solve for X_S :

$$X_S = Q R_S \quad \text{eq. 3.}$$

$$X_S = 0.624 * 36\Omega$$

$$X_S = 22.45\Omega$$

If you'd like to make the network act as a low-pass filter choose a capacitor for X_P ; an inductor for X_P results in a high-pass filter. Remember a capacitor passes more RF and an inductor blocks more RF as the frequency increases. If X_P is a capacitor it shunts more of the higher frequencies to ground and X_S , then being an inductor, blocks more of the higher frequencies from continuing.

Up until now frequency hasn't entered into any of the calculations. The reactance values will work for all frequencies, though their associated component values might become unrealistic at extreme frequencies or for high Q values. For the example being used from

last month the design frequency was 10.1 MHz.

For the low-pass "L-Network", choose C_p to be a capacitor with a reactance of X_p (80.18Ω) at 10.1 MHz. Using equation 4, the reactance formula for capacitors that you learned when you took your ham test:

$$C_P = \frac{1}{2\pi f X_P} \quad \text{eq. 4.}$$

So:

$$C_P = \frac{1}{2 (3.1416)(10.1)(80.18)}$$

$$C_P = \frac{1}{5088} = 0.000197 \mu\text{F}$$

Since f is in MHz, C_p will be in μF . To convert to pF multiply by 1 million:

$$C_P = 197 \text{ pF}$$

The equivalent series circuit of 50Ω in parallel with C_p is a capacitive reactance X_s in series with 36Ω . We've already solved its value as 22.46Ω . To cancel out this capacitive series reactance an inductive series reactance of 22.46Ω is added in series. This is calculated using equation 5, the reactance formula for inductors that you also learned when you took your ham test:

$$L_S = \frac{X_S}{2\pi f} \quad \text{eq. 5.}$$

$$L_S = \frac{22.45}{2(3.1416)(10.1)}$$

$$L_S = \frac{22.45}{63.46} = 0.354 \mu\text{H}$$

The final schematic was shown in figure 1 last month and is redrawn here:

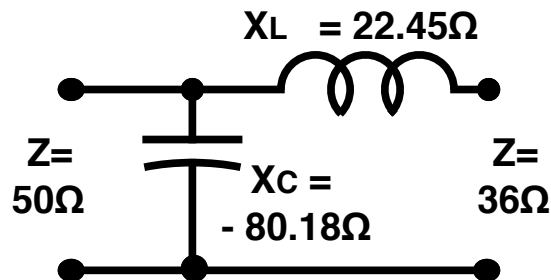


Fig. 1 "L-Network" to Match 50Ω and 36Ω . At 10.1 MHz:
 $C = 197 \text{ pF}$ & $L = 0.354 \mu\text{H}$

Matching Complex Impedances:

So far only purely resistive impedances have been matched. What if the vertical antenna is a little short and has an impedance with a reactive component, say $36 - j10\Omega$? (That's 36 ohms resistance in series with 10 ohms of capacitive reactance.) The answer is simple; since series reactance values add there is now -22.46Ω plus -10Ω or -32.46Ω of capacitive reactance and it must be nulled out with $+32.46\Omega$ of inductive reactance.

Likewise, if the vertical is long and has an impedance of $36 + j10\Omega$? (That's 36 ohms resistance in series with 10 ohms of inductive reactance) it's just as simple; the total reactance is -22.46Ω (capacitive) plus $+10$ ohms (inductive) so only -12.46Ω of capacitive reactance needs to be nulled out with $+12.46\Omega$ of inductive reactance.

And what if the antenna is so long that the inductive reactance is greater than the needed canceling reactance? Perhaps $36 + j30\Omega$? The total series reactance is now $+30\Omega - 22.46\Omega$ or $+7.54\Omega$ inductive. Cancel the series reactance with a 7.54Ω of capacitive reactance. Now the "L-Network" is composed of a shunt capacitor and a series capacitor!

The Hy-Gain Beta Match:

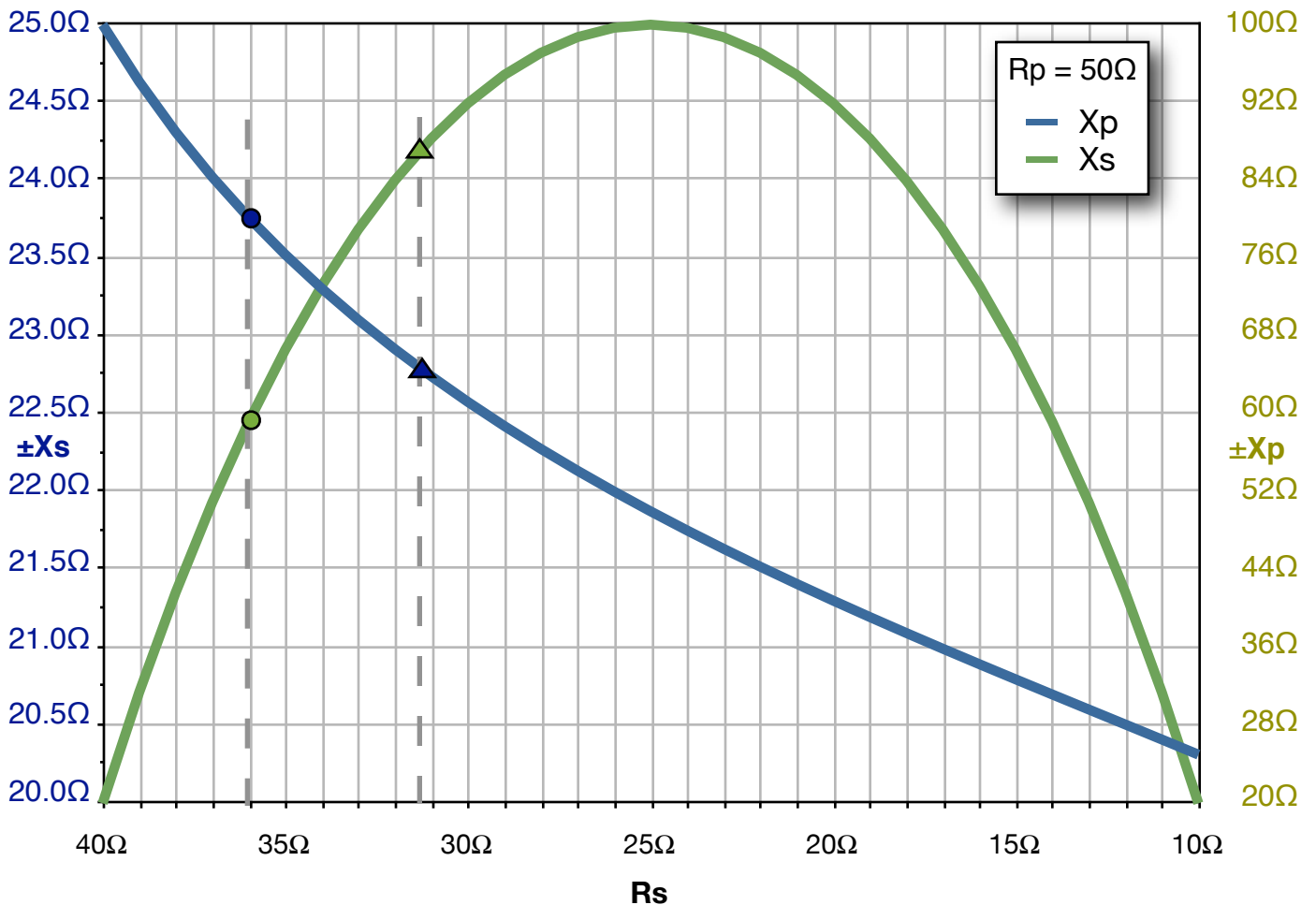
This leads to a clever way to match a beam antenna. The beam's driven element is normally in the range of 10 to 35 ohms depending on the beam configuration. By deliberately making the driven element short, the antenna looks to the feedline like a capacitive reactance in series with the antenna's resistance. This capacitive reactance can be set by adjusting the antenna length to be the value needed to be the series reactance X_s , eliminating a physical capacitor from the antenna design. An inductor is then used to shunt the 50Ω coax side of the antenna to complete what is a high-pass L-Matching network. Hy-Gain uses a length of shorted transmission line stub instead of an inductor and calls this matching design a *Beta Match*. An advantage of the shorted stub is that the

center point of the short can be electrically grounded putting the driven elements at DC ground potential, preventing static buildup and providing some extra lightning protection. Designing this to work on a tri-band beam is a mathematical tribute to the engineers at Hy-Gain.

A Beta match for the vertical:

Figure 2 shows the high pass configuration matching 50Ω to a vertical mounted over perfect ground. The 10.1 MHz vertical is made of 279-9/16 of 1" dia. 6061-T6 aluminum, and has an feedline impedance of $36 + j0\Omega$. When the vertical is shortened its impedance changes; the resistive component goes down and the reactive component becomes capacitive negative). What if we shorten the antenna to the point where the

L-Match Graph: X_s & X_p vs. R_s for 50Ω System



capacitive reactance is the value of X_s needed to match 50Ω to the antenna's new resistive component? This eliminates the physical X_s capacitor and only a shunt inductor is required at the antenna feed-point! This matching technique is the well known Beta Match used by Hy-Gain in many of their antennas.

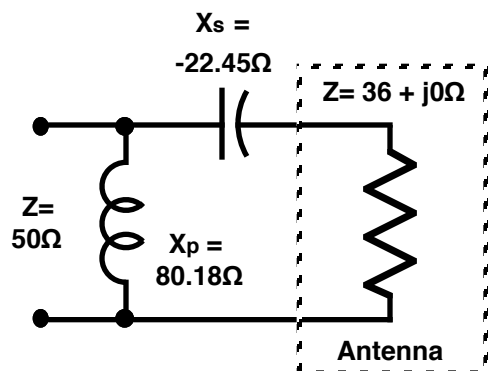


Fig. 2: High-pass "L-Network" matching a "perfect" vertical. At 10.1 MHz: $C = 702$ pF & $L = 1.26$ μ H

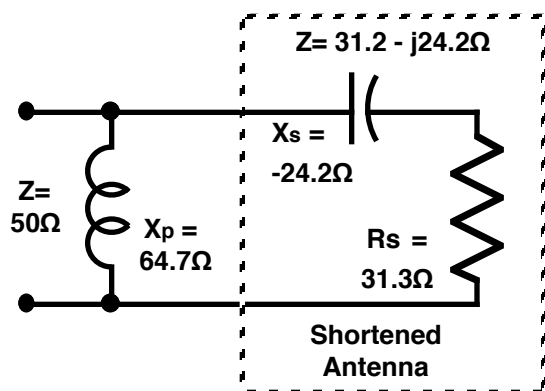


Fig. 3: Beta match of a shortened vertical. X_s is created by shortening the antenna. At 10.1 MHz: $L = 1.02$ μ H

If the vertical is shortened to 266-5/8" the feed-point impedance changes to $31.3 - j24.2\Omega$. Calculating the needed components for an L-Network to match 50Ω (R_p) to 31.3Ω (R_s) are:

$$Q = 0.773 \quad X_s = -24.2\Omega$$

$$X_p = 64.7\Omega \quad L_p = 1.02\mu\text{H}$$

The finished antenna is shown in figure 3; it has a calculated SWR below 1.002:1. Just remember though, this is an antenna of 1" dia. aluminum tubing erected over a perfect ground (lots of radials) and clear of obstacles. Your mileage may vary!

A graph to get you close:

Let's face it, when building a homebrew antenna and winding a coil the coil is not going to be right on on the first try. Once the antenna is built, there will always be some trimming of elements and coils. What is needed is a good starting point. The 50Ω "L-Match" graph gives X_s and X_p for any resistive load between 40 and 10 ohms. The graph is set up for an R_p of 50 ohms and gives you a good starting point and a check for your calculations.

To use the graph, look at the original example – matching a 36Ω vertical to 50Ω feedline. On the horizontal (R_s) axis find 36 ohms. Draw a line vertically from that point. Where the vertical line intersects the green line, read X_s on the left-hand scale (22.4Ω); and where the vertical line intersects the blue line, read X_p on the right-hand scale (80Ω). These points are marked by a small circle.

Another example – matching the shortened antenna to 50Ω feedline – is marked with small triangles. As the antenna was shortened its resistance (R_s) decreased and its reactance (X_s) increased in a negative direction (capacitive). The trick is to find where R_s and X_s fall on the same vertical line. This can be done by trial-and-error, by a lot of math, or by a computer program such as NEC-WIN PLUS, or EZNEC. The result turns out to be 31.3Ω for R_s and 24.2Ω for X_s for our example. Where the vertical line (extended if necessary) passes through the blue

Xp line the correct value of Xp (64.7Ω) can be read on the right scale.

One last thing - The “L-Matching” network is based on the conversion between a parallel impedance and its equivalent series impedance. In the general form this can be done with the following equations:

For converting Parallel to Series *:

$$R_S = \frac{R_P X_P^2}{R_P^2 + X_P^2} \quad \text{eq. 6a.}$$

$$X_S = \frac{R_P^2 X_P}{R_P^2 + X_P^2} \quad \text{eq. 6b.}$$

For converting Series to Parallel *:

$$R_P = \frac{R_S^2 + X_S^2}{R_S} \quad \text{eq. 7a.}$$

$$X_P = \frac{R_S^2 + X_S^2}{X_S} \quad \text{eq. 7b.}$$

“L-Network” Pros and Cons:

The “L-Network works well when matching reasonable impedances. If Q becomes too large or too small then the component values can become unreasonable.

Under moderate to high power operation the voltages across the capacitor can become high. High-voltage and high-Q capacitors that can handle the expected current should be used. If you are unsure, calculate the expected voltage and current, and add a good safety margin. Similarly, coils should be low loss. If the coil's resistance is significant with respect to Rs and Rp then the Q becomes affected resulting in changes to the way the network matches.

If the component(s) of the matching network are out in the open then they need to be protected from the weather. HyGain uses a balance transmission line stub instead of a coil in its antenna designs. This not only allows DC grounding, it also makes for a weather resistant design.

Still, the “L-Network” provides a simple low-loss matching scheme that is easy to calculate and implement. It a method a builder should consider when looking for the best way to match two impedances.

In Conclusion:

If this article has whet your appetite for learning more about antennas, I must recommend that you look into the ARRL Antenna Modeling Course. This class is taught over the Internet. You will need to buy an antenna modeling program and there is a fee for the course. It is a worthwhile expense if you want to know more about antenna design, and how to model an antenna with a computers. You can find more about the course on the ARRL website. Look for ARRL course # EC-004:

<http://www.arrl.org/cce/courses.html#EC004>

* **Notes:** Derivations for equations 1, 6a, 6b, 7a and 7b may be found on our club website at:

http://www.w6ze.org/btt/btt040_notes.pdf

(The derivations are also included with this pdf version of Bob's TechTalk #41.)

73, from AF6C



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Derivations for Bob's TechTalk #41

For the two circuits shown in Fig 1. to be electrically equal the following must be true:

$$R_S + jX_S = \frac{R_P(jX_P)}{R_P + jX_P} \quad (1)$$

This comes from the formula for two impedances in parallel

$$Z_T = \frac{Z_1(Z_2)}{Z_1 + Z_2} \quad (2a)$$

Multiplying the top and bottom of the right side of equation 1 by the conjugate of the denominator yields:

$$\begin{aligned} R_S + jX_S &= \frac{R_P(jX_P)}{R_P + jX_P} \frac{R_P - jX_P}{R_P - jX_P} \\ &= \frac{R_P^2(jX_P) - R_P j^2 X_P^2}{R_P^2 - j^2 X_P^2} \end{aligned}$$

but since $j^2 = -1$, we get:

$$R_S + jX_S = \frac{R_P^2(jX_P) + R_P X_P^2}{R_P^2 + X_P^2}$$

On the right side of the equation, separating the real and imaginary parts yields:

$$R_S + jX_S = \frac{R_P X_P^2}{R_P^2 + X_P^2} + j \frac{R_P^2 X_P}{R_P^2 + X_P^2} \quad (3)$$

Since the real and imaginary parts of equation 4 must be equal, we get:

$$R_S = \frac{R_P X_P^2}{R_P^2 + X_P^2} \quad (\text{eq. 6a})$$

$$X_S = \frac{R_P^2 X_P}{R_P^2 + X_P^2} \quad (\text{eq. 6b})$$

Going from parallel to series we can rewrite equation 2 as:

$$\frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{Z_T} \quad (2b)$$

Substituting the components in figure 1 we get:

$$\frac{1}{R_P} + \frac{1}{jX_P} = \frac{1}{R_S + jX_S} \quad (4)$$

Multiplying the top and bottom of the right side of equation 4 by the conjugate of the denominator yields:

$$\begin{aligned} \frac{1}{R_P} + \frac{1}{jX_P} &= \frac{1}{R_S + jX_S} \frac{R_S - jX_S}{R_S - jX_S} \\ &= \frac{R_S - jX_S}{R_S^2 - j^2 X_S^2} \end{aligned}$$

Using identities $\frac{1}{j} = -j$ and $j^2 = -1$ yields:

$$\frac{1}{R_P} - j \frac{1}{X_P} = \frac{R_S - jX_S}{R_S^2 + X_S^2}$$

On the right side of the equation, separating the real and imaginary parts yields:

$$\frac{1}{R_P} - j \frac{1}{X_P} = \frac{R_S}{R_S^2 + X_S^2} - j \frac{X_S}{R_S^2 + X_S^2} \quad (5)$$

Since the real and imaginary parts of equation 4 must be equal, we get:

$$R_P = \frac{R_S^2 + X_S^2}{R_S} \quad (\text{eq. 7a})$$

$$X_P = \frac{R_S^2 + X_S^2}{X_S} \quad (\text{eq. 7b})$$

$$Q = \sqrt{\left(\frac{R_P}{R_S} - 1\right)} \quad (\text{eq. 1})$$

Equations 2 and 3 are definitions of Q for a series and parallel circuit. Equations 4 and 5 any ham who passed his/her test already knows. So that leave us to prove equation 1 from the article.

Of course, there are numerous other ways to derive these equations, but this follows most closely the theme of the original article by not using admittance nor vector representation of the impedances.

We'll start with (eq. 6a):

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$$R_S = \frac{R_P X_P^2}{R_P^2 + X_P^2}$$

dividing the numerator and denominator by: X_P^2 yields:

$$\begin{aligned} R_S &= \frac{R_P}{\frac{R_P^2}{X_P^2} + \frac{X_P^2}{X_P^2}} \\ &= \frac{R_P}{\left(\frac{R_P}{X_P}\right)^2 + 1} \end{aligned}$$

Rearranging equation eq. 2 from the article results in:

$$Q = \frac{R_P}{X_P} \quad (\text{eq. 2b})$$

Substituting Q into the equation and then solving for Q yields:

$$\begin{aligned} R_S &= \frac{R_P}{Q^2 + 1} \\ Q^2 &= \frac{R_P}{R_S} - 1 \end{aligned}$$