BOB'S TECHTALK

Number 52:
The MILLER EFFECT and the CASCODE AMPLIFIER
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Introduction:

The Miller Effect is an electronic principle where the capacitance at the input of a common cathode triode amplifier increases with the gain of the amplifier. If the impedance driving the amplifier is high, then the increased capacitance can reduce the bandwidth, especially at high frequencies. There are ways to reduce the Miller effect, one being the use of a cascode amplifier.

In the Heathkit of the Month #91 article both the Miller Effect and the Cascode amplifier are mentioned. Here are explanations of what the Miller Effect is and what the Cascode Amplifier can do to reduce its effect.

The Miller Effect:

The Miller Effect was documented by John Milton Miller (1888 - 1968). while he was experimenting with triode vacuum tubes. His distinguished career is well documented on Wikipedia. Miller published a paper in 1920 on the effect later named for him. While early on, the Miller Effect was not a problem in most circuits, as frequencies increased the added capacitance detracted from the gain and bandwidth. Today, the Miller Effect also occurs with bipolar and field effect transistor amplifiers.

Figure 1 shows a simple triode vacuum tube amplifier. An AC signal from the generator (*GEN*) is coupled to the grid of the triode with capacitor C_{IN} and the cathode is bypassed with capacitor C_K. Assume both of these capacitors are large enough to have little, if any, effect on the circuit. R_1, R_2 and R_K set the bias for the triode amplifier. C_{gp} and C_{gk} represent the in-

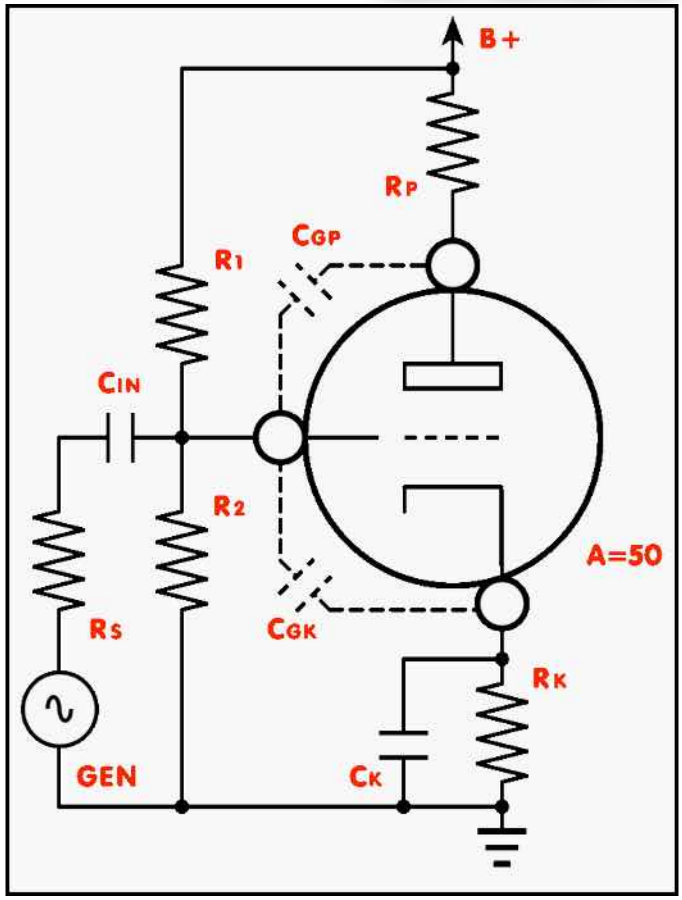

Figure 1: Typical small signal triode amplifier.

ternal capacitance between the grid and plate, and the grid and cathode, respectively. Let's assume the circuit has a gain of 50. *GEN* and R_s represent the signal source and source resistance; this usually represents the previous stage or the antenna (or other) input circuit.

Figure 2 is an AC equivalent of Figure 1. C_K and C_{IN}, being large at the input frequency, are treated as shorts. Assuming the input is a signal of 10 mV peak-to-peak (P-P), and the amplifier gain is 50, a 500 mV P-P signal appears at the plate of the tube. A common cathode amplifier creates a signal at the plate that is 180° out of phase with the signal at the grid (When the grid is at the negative peak of its cycle, the signal at the plate is at its positive peak.)

If you look at C_{gp} you'll notice that when the voltage on the grid side changes, the voltage on the plate side changes 50 times more, and in the opposite direction. At the grid terminal this

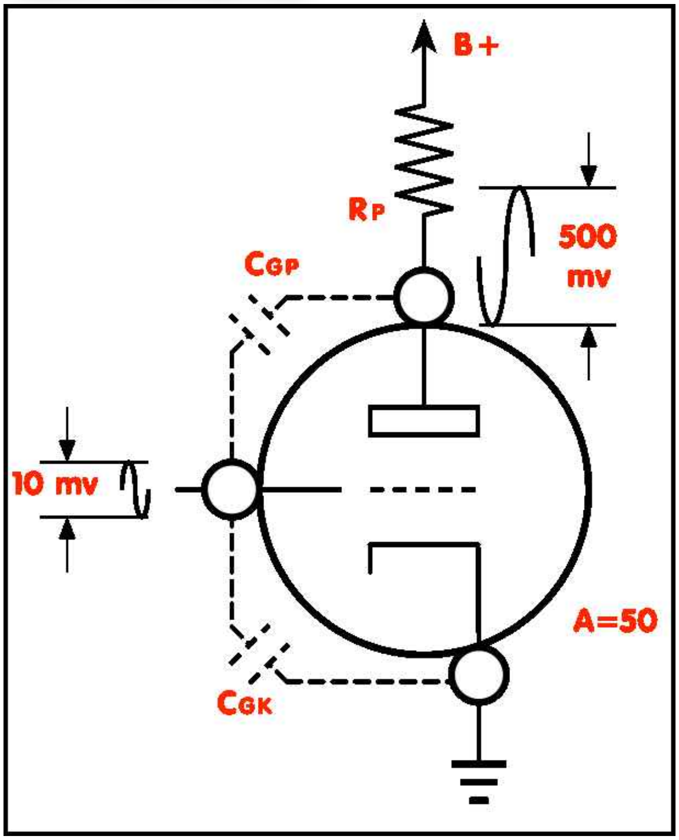

Figure 2: The AC equivalent of Figure 1 treating CK and CIN as shorts.

makes the effective grid capacitance (C_{eff}) much larger than the C_{gp} value. The equation is:

$$C_{eff} = C_{gk} + (1 + |A|)C_{gp} \qquad \text{eq. 1}$$

Where A is the actual amplifier gain. If the cathode resistor is reasonably well bypassed C_{GK} doesn't change. The vertical bars on either side of A in the equations signify "the absolute value" (i.e. treat A as always a positive value).

Typical values for triodes for C_{gp} and C_{GK} are between 1.4 and 2 pF and 1.6 and 4 pf respectively. These seem like pretty small values , but with the Miller Effect C_{eff} will increase that value 10 to possibly 100 fold. If the source impedance is 5 KΩ and the Miller capacitance is 15 pF, the amplifier gain will be down 3 dB at a mere 2.1 MHz.

Solutions for the Miller Effect:

By placing another grid that has a low AC impedance (Capacitively well biased to ground at the operating frequencies) between the control grid and plate the Miller Effect can be reduced substantially. Yet, the two-grid tetrode introduces problems of its own, secondary emission; the solution is to add a third grid which results in the common pentode tube. At audio and HF radio frequencies the pentode performs very well and was used heavily throughout the electronic industry in the vacuum tube era. It does have some drawbacks though, - at VHF and UHF frequencies it becomes noisy. This noise is not noticeable at HF frequencies because it is masked by the prevalent external noise. Triodes are a lot quieter, but not good at those higher frequencies due to the Miller Effect. A solution is the cascode amplifier.

If you are not into math, feel free to skip to the section titled "The Cascode Ampifier"

Some Basic Vacuum Tube Information:

On data sheets the maximum gain of a triode tube is usually expressed by its amplification factor (mu or μ). The gain of a common cathode triode amplifier is given as:

$$A_{CK} = \mu \frac{R_L}{r_P + R_L} \qquad \text{eq. 2}$$

Where A_{CK} is the actual gain, R_L is the load resistance, which is R_P in parallel with the load of the next stage, and r_P is the tube plate resistance (μ and r_P are given on the tube data sheet). Note that A can never be larger than μ, thus μ, the amplification factor, is the largest possible gain of a specific triode. Variables are listed and defined in **Table I**.

Amplification factor μ is determined by measuring the plate current change with a small change in grid voltage, then holding the grid voltage constant at its initial value and increasing the plate voltage until the same increase in plate current is reached. The amplification factor is the second voltage change divided by the first voltage change:

$$\mu = \frac{\Delta V_P}{\Delta V_G} \text{ for the same } I_P \qquad \text{eq. 3}$$

As an example, a triode is biased for nominal operation. The grid voltage is then increased 250 mV and the grid current increases by 4 mA. Next, the grid voltage is restored to its initial value and the plate voltage is raised until the plate current increases by 4 mA. The plate voltage increase is then noted, and say it's 4.5 volts. The amplification factor (μ) is then 4.5 V divided by 0.25 V or 18. μ has no units; it is a pure number or ratio. Since a vacuum tube isn't linear over its operating range, μ can vary somewhat depending on the initial bias point.

Plate resistance r_P is determined by the amount the plate current changes with a change in plate voltage with the grid voltage held constant:

$$r_P = \frac{\Delta V_p}{\Delta I_p} \quad \text{for the same } V_G \qquad \text{eq. 4}$$

Transconductance (g_m) is another parameter of a vacuum tube that is used in calculating gain. It is more commonly associated with tetrodes and pentodes since their plate current changes much less with changes in plate voltage due the action of the other grids. Thus they have extremely high values of μ, but also high values of r_P making gain calculations using μ problematic. Transconductance is a measure of the change in plate current with a change in grid voltage, with the plate voltage held constant:

$$g_m = \frac{\Delta I_P}{\Delta V_G} \quad \text{for the same } V_P \qquad \text{eq. 5}$$

Since g_m is current divided by voltage it is conductance, the reciprocal of resistance and is measured in mhos. Since g_m is a small number it is commonly specified in µmhos.

The gain of a common cathode tube amplifier, expressed by its transconductance is:

$$A_{CK} = g_m \frac{r_P R_L}{r_P + R_L} \qquad \text{eq. 6}$$

Comparing eq. 6 with eq. 2 you get:

$$g_m \approx \frac{\mu}{r_P} \qquad \text{eq. 7}$$

Note the approximate equal (≈) notation. Since μ and g_m are determined by different means, the equation is a first order approximation, but still reasonably close. If r_P is much greater than R_L then eq. 6 becomes:

$$A_{CK} \approx g_m R_L \qquad \text{eq. 8}$$

A common grid amplifier is also part of the cascode amplifier. This amplifier offers low input resistance and gain similar to the common cathode amplifier:

$$R_{IN} \approx \frac{1}{g_m} \qquad \text{eq. 9}$$

$$A_{CG} \approx g_m R_L \qquad \text{eq. 10}$$

The Cascode Ampifier:

Figure 3 shows a basic cascode amplifier. The word cascode comes from the fact the circuit uses two ***casc***ading triodes to act as a pent***ode.*** V1 is a common cathode amplifier, but instead of a load resistor, the plate is connected to the input of a common grid amplifier, V2, whose input resistance is given in eq. 9. The gain of the V1 circuit is given in eq. 8, with R_L being R_{IN} . This math is simple, assuming the two tubes are similar so the g_m of the two tubes are the same:

$$A_{V1} \approx g_m \left(\frac{1}{g_m} \right) = 1 \qquad \text{eq. 11}$$

The common grid amplifier V2 has a gain as given in eq. 10 where R_L is the V2 plate resistor in parallel with whatever the load resistance of the next stage is. The total gain of the cascode amplifier is approximately identical to the common cathode amplifier. What has changed is the input capacitance of the amplifier.

Since the gain of the first stage is now one, the capacitance due to the Miller Effect (eq. 1) is just:

$$C_{eff} = C_{gk} + 2C_{gp}$$

This results in a lot lower input capacitance and results in much higher frequency response. Since the common grid amplifier V2 has no signal on its grid, only DC bias with any AC components shunted to ground by the grid bypass capacitor, there is no Miller Effect associated with V2.

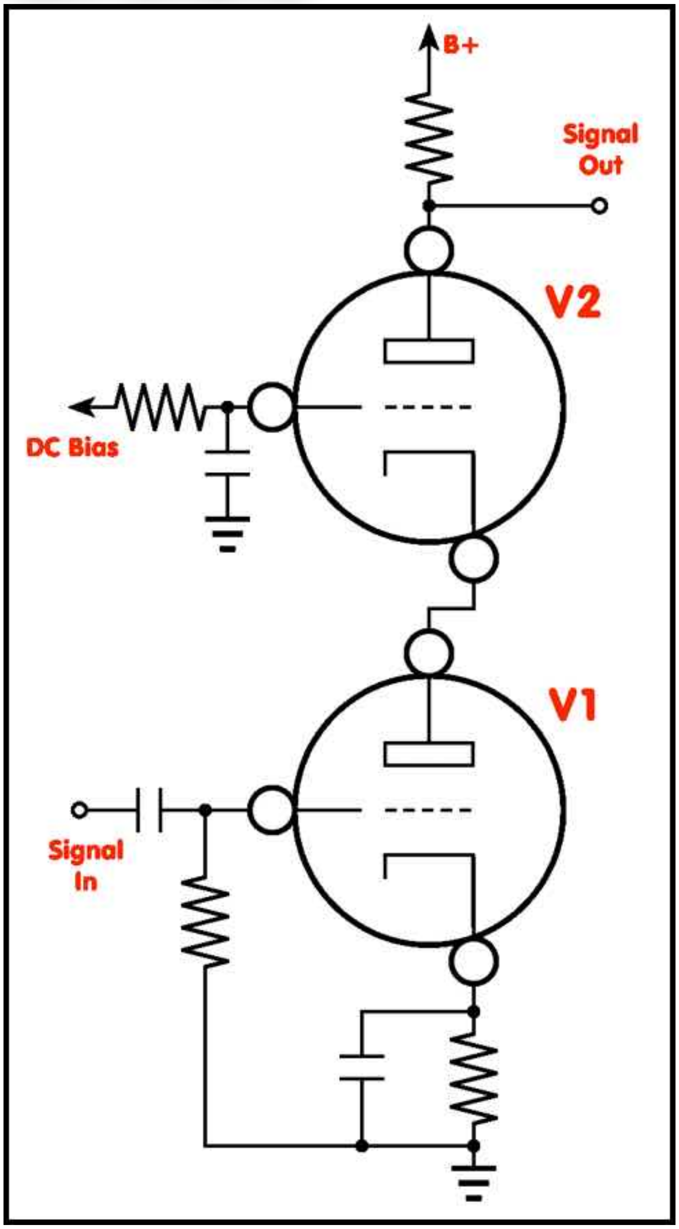

Figure 3: Basic cascode amplifier circuit.

Comments:

While a pentode, with its much lower grid to plate capacitance would also provide low input impedance and possibly even more gain, it will also tend to generate more noise. While at HF frequencies this noise would be indistinguishable from atmospheric noise, at VHF and UHF frequencies this would not be the case without using a more expensive vacuum tube and additional circuitry.

The cascode amplifier is made even more practical due to the availability of dual triodes in a single envelope eliminating the need for a second tube socket, and assuring the two triodes are similar. The added bias for V2 is no more complex than supplying screen voltage to a pentode, and can be tapped off the connection between the the plate of V1 and the cathode of V2, with a suitable resistor and bypass capacitor.

73, from AF6C

Mathematical Variable & Figure Description Table

A_{CG}	Common grid amplifier gain. See eq 10.
A_{CK}	Common cathode amplifier gain. See eqs. 2, 6 and 8 and associated text.
A_{V1}	Gain of the first cascode triode.
C_{eff}	Effective grid capacitance as defined by eq. 1
C_{gk}	Vacuum tube interelectrode capacitance between the grid and the cathode.
C_{gp}	Vacuum tube interelectrode capacitance between the grid and the plate.
C_K	Cathode bypass capacitor to ground.
C_{IN}	Grid input coupling capacitor.
g_m	Transconductance as defined by eq. 5.
I_P	Plate current.
R_1, R_2	Typical biasing resistors.
R_K	Cathode biasing resistor.
R_L	Plate load resistance. Usually R_P in parallel with the load imposed by the next stage.
r_P	Tube plate resistance defined by eq.4.
R_P	Plate resistor (usually to B+).
R_S	Input source resistance. In Fig. 1 the output resistance of GEN.
μ	Amplification factor as defined by eq. 3.
V_G	Grid voltage.
V_P	Plate voltage.

Table I

This article is based on the TechTalk article that originally appeared in the May 2019 issue of RF, the newsletter of the Orange County Amateur Radio Club - W6ZE